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A Newly Found Letter of Al-Kāshī on Scientific Life in Samarkand

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The famous 15th-century Iranian mathematician and astronomer, Jamshīd al-Kāshī, left his native Kāshān for Samarkand in order to participate in the scientific activity there, upon Ulugh Beg's invitation. Al-Kāshī corresponded in Persian with his father, who lived in Kāshān. One of his letters to his father was published and translated by Kennedy (1960) and Sayılı (1960). In this paper, we present an English translation with commentary of another letter of al-Kāshī to his father, which has been found recently in Iran. Like the previous one, this new letter contains interesting information on Ulugh Beg's scientific circle in Samarkand. © 1997 Academic Press

چکیده

جمشید کاشانی ریاضیدان و اختر شناس نامدار ایرانی در قرن نهم هجری، به دعوت الغ بیگ برای شرکت در فعالیت علمی از شهر خود کاشان به سمرقند رفت. کاشانی با پدرش که در کاشان می زیست به فارسی مکاتبه می کرد. یکی از نامه های او به پدرش قبلاً شناخته شده و به زبانهای مختلف ترجمه، شرح و منتشر شده است. در این مقاله، نامهٔ دیگری از جمشید کاشانی که اخیراً یافته شده معرفی می شود. این نامهٔ نویافته همانند نامه ای که قبلاً شناخته شده بود حاوی اطلاعات جالبی در بارهٔ محفل علمی الغ بیگ در سمرقند است.

الملخّص

انتقل غياث الدين جمشيد الكاشي الرياضي و المنجّم الايراني الشهير في قرن التاسع من الهجرة، من مسقط رأسه كاشان الي سمرقند، للمساهمة في النشاطات العلميّة بطلب الغ بيكٍ منه. كان الكاشاني يراسل اباه باللغة الفارسية و كانت احدي رسالاته الي ابيه معروفة من قبل، و قد ترجمت الي لغاتٍ اخري مع شيءٍ من الشرح، و طبعت. في هذه المقالة يعرّف رسالة اخري منه الي ابيه، حيث وجدت في السنوات الاخيره. تحتوي هذه الرسالة التي وجدت اخيراً – كسابقتها الّتي كان معروفة – علي معلومات فريدةٍ حول الحلقة العلمية المؤسّسة علي يد الغ بيك.

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INTRODUCTION

Ghiyāth al-Dīn Jamshīd b. Mas'ūd b. Maḥmūd Kāshānī, better known in Western literature as al-Kāshī, was a prominent Iranian mathematician and astronomer of the medieval Islamic world. He was born in Kāshān, where he was engaged in astronomical observations before his departure for Samarkand in 1421 (824 A.H.). He died in 1429 (832 A.H.) [10, 7:255–262; 16, 2:480–486; 18, 8].

Al-Kāshī joined the scientific circle of Ulugh Beg, the ruler of Samarkand, and was supported by him. He corresponded in Persian with his father, who lived in Kāshān and who apparently was familiar with mathematics and astronomy. One letter from al-Kāshī to his father was translated into English independently by Kennedy [12] and Sayılı [21] (with the Persian text). Russian, Arabic, Turkish, and Uzbek translations have appeared as well.

In this letter, al-Kāshī says that he repeats some stories about people's opinion of him from an earlier letter which he had sent to his father "through the merchants of Qum" [12, 193; 21, 93]. In the winter of 1994, I found another letter from al-Kāshī to his father in the Majlis Library, Teheran, in MS. No. 5138/142. This manuscript is different from the letter published in [12; 21], but al-Kāshī's description of people's opinion of him is similar to that in the published letter. Thus the manuscript must be a copy of the first letter, which was hitherto believed to be lost. From now on we will call the letter published in [12] and [21] the second letter. The first letter includes much new information, and even in the overlaps we find details which help clear up some phraseological or topical ambiguities in the second letter. The unique manuscript of the first letter is followed by a hitherto unknown manuscript of the second letter.

The first letter was written about two years after al-Kāshī's arrival in Samarkand (see line **34** below), i.e., around A.D. 1423. The manuscript text of the letter has 80 lines. The numbers of the lines are indicated in boldface in the following English translation, for easy reference to the contents of the letter. The final section of the letter dealing solely with prosody (lines **64** to **79**) is omitted here, because the contents are complicated and difficult to translate into English. This part and the illegible words in line **62** are shown as dots in brackets. The same notation is used for the location of an apparent lacuna in line **45**. My own explanatory additions to the text appear in square brackets. An asterisk (*) in the translation refers to the commentary.

The manuscript is not an autograph, and some errors in the text seem to be due to the scribe's carelessness. The prolixity of the composition, especially in the preamble of the letter, has been avoided in the translation as far as possible. Where the exact meaning of a phrase was not clear to me, I have chosen the most probable one. The complete Persian text of the first letter with commentary, including the section on prosody, is published in [1].

I am glad to publish this extremely valuable document which—like the second letter—provides historians of science with a first-hand source about the scientific atmosphere of Samarkand in Ulugh Beg's time.

TRANSLATION OF AL-KĀSHĪ'S FIRST LETTER TO HIS FATHER

(1) [Following is] the text of the letter of our master, the greatest pride of philosophers [or sages, *ḥukamā'*] and geometers, *Maulānā* (*) Ghiyāth al-Dīn Jamshīd al-Kāshānī, written to his father in Kāshān from the capital Samarkand while he was engaged in astronomical observation (*raṣad*) (*) there.

The supreme ordinance, with which one must comply, issuing forth from [my] real master and overall support [i.e., you my father], may Almighty God keep his fatherly **(2)** shadows [cast on me], honored [me] [i.e., the letter arrived] at the most propitious time and the most auspicious hour for [its] arrival. To praise God for that, I (*) performed [devotional] prostrations for praising the Almighty Creator. [Receiving and reading it] produced [in me] all kinds of delight and joy, and because it comprised [the good news of] the good health of the august person of my master and [His] infinite blessings, I thanked and praised [God, and I made all] kinds of pleas **(3)** and entreaties.

Verily, and I swear by the greatness of Almighty God, [my] ardent desire and longing for the honor of kissing the hand of my master are infinite. Invoking God, and pleading with Him by the rightfulness of "him after whom there will be no other prophet" [i.e., the prophet Muḥammad] (*), I beg the Bestower of Gifts [i.e., God] for a grace which would include [my] attaining that felicity.

Then, [let us consider] the advice which you have given me **(4)** to the effect that I ought not to occupy [my] mind with other arts because I am engaged in the important matter of astronomical observations. You have made this interdiction for two reasons: first, because occupation with another subject would indeed distract [me] from the astronomical observations; second, because of [my] occupation with another art in which I may be a beginner, there may occur in [my] discussions or compositions some defect or error **(5)** which people would bring to bear on the other arts [in which I am adept].

The situation in the city of Samarkand, may God preserve it from adversities, is [as follows]. His Royal Majesty (*) [i.e., Ulugh Beg (*)] had donated a charitable gift [*ṣadaqa*] amounting to thirty thousand *kopakī* (*) dinars, of which ten thousand had been ordered to be given to students. [The names of the recipients] were written down: [thus] ten thousand-odd students steadily engaged in learning and teaching, and qualifying for a financial aid, were listed. **(6)** There are the same number [of students] among the notables and their sons, who dwell in their own homes. Among them there are five hundred persons who have begun [to study] mathematics. His Royal Majesty the World-Conqueror, may God perpetuate his reign, has been engaged in this art [i.e., mathematics] for the last twelve years. Students, too, are indeed inclined to it **(7)** and are working hard on it; [in fact,] they are trying their hardest. This art is taught at twelve places—a number inferior to that of [mathematics] teachers. Thus, nowadays [the state of teaching and learning mathematics in Samarkand] has no parallel in Fārs [i.e., Persia, the southern province of Iran] and 'Irāq [i.e., the western part of modern Iran, the ancient Media]. There are twenty-four calculators [*mustakhrij*], some of whom are also astronomers and some have begun [studying] Euclid ['s *Elements*].

Well, when **(8)** I arrived here, as it is customary [everywhere], all of them began inquiring and spying in order to find out the extent of my knowledge [lit., "beginning"] in this art, so that, if they found me to have little knowledge or to be equal or slightly superior to them, they keep their positions; thus they might not [have to] say that a stranger came in and got the better of them. During the period that the arena was theirs, **(9)** in the discussions held in the presence of His Royal Majesty they were confronted with some difficulties into which they had looked for a month or two or even for a year, but to which no solution had been found. For example, this problem: [Let us suppose] somebody is standing on a perfectly circular ground or on the sea surface, and the visual ray issuing from his eyes is tangent **(10)** to that, and [then] reaches the sphere of the ecliptic [*falak al-burūj*, i.e. the outermost sphere of the universe]. Now, at which distance will [that ray] intersect the true horizon, and, where it reaches the sphere of the ecliptic, how much will it be depressed from the true horizon? (*) And many other [problems] which were detailed [to you] previously.

[Well,] the very day I reached Samarkand, although my baggage had not arrived yet and I had no book or *zīj* [set of astronomical tables] with me, **(11)** I borrowed a reed pen and ink-pot from a science student in a school, solved all [those problems], and submitted [my solutions] to His August Majesty. [Later,] His Majesty put me to the test in similar cases. [For instance,] I was ordered to set up a gnomon [*miqyās*] on a wall of the royal palace, the surface of which [wall] is not in alignment either with the meridian plane or with the initial plane of azimuths [i.e., the East–West plane], and to draw the lines of equal [i.e., equinoctial] and **(12)** unequal [i.e., seasonal] hours on it, that is to say, a sundial [*rukhāma*] on the surface of a wall of unknown [i.e., arbitrary] azimuth. I drew them (*). All the specialists in [this] art came to view [the sundial], [including] Qāżīzāda Rūmī (*), who is the most learned of all, and who confessed that this [scheme] was extremely difficult. He tried in vain to figure out the proof thereof, and finally asked me about it, **(13)** [thus] acknowledging my victory in the presence of many people. After examining [the sundial] in different ways, he avowed that [the scheme] was really well thought out. Similarly, His Majesty [once] said: "We would like to make a hole in the wall of a *miḥrāb* [prayer niche in a mosque] in such a way that the sun may shine through that hole for a short while at the afternoon [prayer] time both in summer and in winter. That single hole **(14)** must be round from inside, but from the outside it must be in such a way that sunshine cannot pass through it at times other than the afternoon [prayer time]. This [royal wish] had been [already] expressed before my arrival, and nobody had been able to realize it; [but] when I came [here], I did this also. (*)

Further, His Royal Majesty visits the school once every few days, and **(15)** attends a class for some time, and I am in His Majesty's company. [On these occasions] we do not know at all what the lesson will be about, nor what problem will be brought up, [whereas] the teacher and students have studied it together the night before. I have [sometimes] intervened without preparation in a [discussion] and I have spoken so much about the point [in question] that everyone was astonished.

(16) One day, His Majesty was reading a treatise in which a reference was made to *al-Qānūn al-Mas'ūdī* ("The Mas'ūdic Canon"), a work composed by Abū Rayḥān [al-Bīrūnī] (*) like the *Almagest*. He sent for the *Qānūn*, in which he found the problem; but because of some difficulty [in the problem], it was not understood. I happened to have quotidian fever for two days, and did not present **(17)** myself [at the court]. Qāżīzāda had been there. [His Majesty] had placed the *Qānūn* manuscript before him. [The problem] could not be solved then and there; [so] Qāżīzāda had taken [the manuscript] to his *wuthāq* [cell or chamber in a *madrasa*, i.e., school], and had meditated upon [the problem], but in vain. On the third day he had taken [the manuscript] back [to His Majesty], saying: "Certainly there is at this point a lacuna left by the scribe, for the problem is not fully solvable." At this point I arrived there. His Majesty, may God perpetuate his reign and sovereignty, pointed to me and **(18)** said: "There is a difficulty at this point. Think it over." And he passed the manuscript on to me with his blessed hand. I reflected awhile on the manuscript, found out [the solution], and submitted it immediately to His Majesty, who repeatedly expressed his admiration for me.

Another day, when arriving at the school, His Majesty had met a student at the door, **(19)** holding a book. He [i.e., the King] had asked him what book it was. [The student,] kissing the book, had presented it [to His Majesty]. Opening the book, [His Majesty] had chanced on a chapter [entitled] "On the curiosities of the astrolabe," [beginning with this problem]: "[Let us suppose that] the Sun is, e.g., in 10 degrees of Aquarius, with a certain altitude, and the ascendent of time is a certain degree [of the ecliptic]; then [the ascendent of the time when] its [i.e., the Sun's] altitude [is the maximum altitude of the ecliptic at that moment] is a quadrant [in advance of the Sun's position], i.e., in 10 degrees **(20)** of Taurus. After one month, the Sun having described one sign [of the Zodiac], while having the same altitude as on that [previous] day, how could the ascendent be exactly the same as it was in that day?" (*) After having entered [the classroom, His Majesty] had presented that problem for discussion. Someone said that this was most likely where the latitude of the city exceeds the [complement of the (?)] [sun's] declinations [i.e., plus or minus 67.5 degrees], and the signs rise inversely. They had **(21)** thought about it for a long time. I arrived there towards the end of the meeting. [His Majesty] immediately ordered the book to be handed to me, saying: "Figure out where this [case] could be." As soon as I had a look [at the problem], I said it could be at Samarkand, and I gave reasons [for my statement]. Some students did not **(22)** understand. [So] an astrolabe was brought, and I made [my point] clear to them. [Well,] what I mean is that [His Majesty] has recognized my competence after witnessing all that and many similar cases rather than by conjecture or on the authority of others.

Concerning the observatory, at first [His Majesty] had imagined that Ptolemy's method should be faithfully followed. [Consequently] he had ordered two rings [to be made], each 6 **(23)** cubits (*gaz*) in diameter and 2 digits (*iṣba'*) in thickness. He was unaware [of the fact] that later [scholars] had discovered [new] subtle points [i.e., improvements] and had made more precise [instruments]. [For instance,] he

certainly did not know about the quality of the Marāgha observatory. [When] I came, I told [him] that in the observatory constructed in 'Aẓud al-Daula's time the ring made for it was **(24)** 10 cubits in diameter and one span of the hand in thickness so that the ring might not be distorted; and that, however, subsequent astronomers had turned away from it because of some defects inherent in it, and had relied on the *Fakhrī* sextant invented in Fakhr al-Daula's time. I explained the cause of those defects, and [I added that] the same *Fakhrī* sextant had been constructed at the Marāgha observatory. **(25)** Because His Majesty had seen the internal parts of the latter in his childhood, as soon as I pointed out those parts to him, he remembered them, and said that I was right and that was so. [Thus] he made up his mind [that] the rings which had been cast were decidedly of no use, and that they were not even worth being completed as armillary spheres, for they were too thin. [So] those rings were [ordered to be] broken, **(26)** and an observatory like that of Marāgha was founded as I had suggested. However, an innovation has been made in the position of the *Fakhrī* sextant: The building was designed in circular form with a perimeter of 200 cubits of Kāshān; it stands on top of a rock, in which part of the sextant was carved, so that the edifice might not be very tall, because, bricks not being firm enough here, **(27)** excessive height of a building may cause fracture. At Marāgha, the sextant is positioned higher [but] the other [compartments of the observatory] are not so high, [which is] a bad configuration. Here, the surface of the roof, too, will be flat, so that other astronomical instruments may be placed on it. (*)

Moreover, there are three teachers in the school of His Majesty, may God perpetuate his reign and sovereignty: (1) Qāżīzāda, (2) *Maulānā* Muḥammad Khānī (*), [who] **(28)** surpasses the others by far in sciences other than mathematics; he is endowed with a phenomenal memory, so he teaches from memory most science lessons, and he studied astronomy with Sayyid Sharīf (*); (3) *Maulānā* Abu'l-Fatḥ (*), whom His Majesty met in Herāt, who is versed in Islamic jurisprudence, and has composed [a tract on] astrolabe reading. He teaches only the latter subject. In short, *Maulānā* Muḥammad Khānī, **(29)** when occasionally being present [at the court] or meeting [His Majesty] at the observatory site, has repeatedly said in all fairness: "I used to be singled out in Samarkand for my knowledge of astronomy, and nobody could compete with me. Now, Qāżīzāda, having practiced a lot, has surpassed me, and *Maulānā* Ghiyāth al-Dīn accomplishes miracles of presence of mind. To do them justice, [I must admit that] **(30)** when there is a discussion in their meeting, I dare not intervene, all the more so because His Majesty knows this art well and [therefore] one cannot impudently claim competence. The subject of the problem [to be submitted in the session] is already known [to the students; therefore], presence of mind is a must, and we [i.e., I] happen to lack presence of mind in this art. All night long we study a lesson and the next morning we teach it with a thousand artifices. But there is no need for extensive studying in other sciences."

(31) By annoying you [with these details], I intend [to let you know] that I have entered a field where my colleagues in this art are Avicennas [i.e., top experts] in

other sciences. [However,] on account of [my] presence of mind and of my investigations in all branches of this art, I have come to a point where His Majesty the World-Conqueror, may God perpetuate his reign and sovereignty until the Day of Resurrection, has repeatedly said in assemblies and **(32)** gatherings: "In this art *Maulānā* Ghiyāth al-Dīn's presence of mind is greater than Qāżīzāda's, and his knowledge and intellect are superior to the latter's, so much so that *Maulānā* Ghiyāth al-Dīn solves immediately any difficult problem that comes up, [whereas] Qāżīzāda thinks it over for several days and fails **(33)** to solve it." Similarly, I have adopted here such a way of life that His Majesty, may God perpetuate his reign and sovereignty, has said: "He is really a nice man, and has a decent mode of life. No matter which one of the *mullā*s [i.e., learned men] who came to us, for example, such and such a person, after we gave him sympathetic consideration for a week, he became so conceited that he began **(34)** quarreling with people and committing indiscretions. *Maulānā* Ghiyāth al-Dīn has been here for two years now. We have given him all manners of regard and solicitude; we converse with him every day, and we have often expressed our admiration for him. Anyone else would have started indiscretions. During all this time he certainly has not quarreled with anybody or **(35)** complained about anybody, and no one has ever complained about him. He did not tell on anybody in our presence out of ambition and covetousness. He does not mix with anybody; he is always busy with [his] work. He is a nice fellow." I thank the exalted and sanctified Creator for these two [royal considerations], because people constantly endeavor to go about both the acquisition of arts and [adoption of] a mode of life in such a way that they may be admired by their neighbors **(36)** and colleagues of the same class. By the grace and success granted by Almighty God, I behaved in a way that such a learned and talented king admires [me] in both regards. "That is the bounty of God; [He gives it to] whom He will." (*) As for one's livelihood, it is [generally] admitted that nobody knows about it better than kings.

[Now] I will mention the King's skills [not just] (*) by way of courtesy. First, he knows Arabic syntax well, **(37)** and writes [Arabic] elegantly. He knows Arabic, Persian, Turkish, Mongolian and some Chinese. He knows by heart all the glorious Koran, and knows well the commentary and citations thereof, thereby making apt quotations [therefrom] when speaking. He has memorized all the dates [for example, of events which occurred during his life]. He knows the art of prosody and versification extremely well. He knows by heart Anwarī's (*) whole divan and some poems **(38)** by Ẓahīr Fāryābī (*). He occasionally composes excellent poems after Anwarī. I do not remember anything [of his poems] to write down [here].

His fantastic memory is one of the wonders of the world. For instance, one day he sat down and began recounting all the places and dates of his residence since he had been in Sulṭāniyya, telling, [for example,] what day **(39)** of the week and the month it was when he [and his party] left such and such locality, halted for noon prayers at such and such place, and leaving there, when and where they traveled by night, and where they halted next. He [thus] came to the point of narrating what the envoys and spies had reported [to him during the trek]. He had

the diaries [of that long journey] brought to him [and read out]. We were listening. [The diaries] accorded [with his narrative]. [Subsequent events] down to that day when we were at that **(40)** meeting [had also been recorded]. Those [details] which had been recorded accorded with what he had [just] said. I had recorded some of those days in [my] diary—for example, the dates of my reaching Herāt, departing [thence] for Samarkand, and moving from garden to garden in Samarkand. When he came down to these [events], I took up [my] diary; on the whole, [what he related] was in accordance [with my records] **(41)** as to the day of the week and the month, and he recounted most of the things that had happened on a [particular] day.

He has great skill in mathematics. For instance, once when he was out hunting, while on horseback he wanted to determine [exactly] when in summer or winter had occurred an event which was known to have taken place on a Monday between the 10th and 15th of Rajab 819 [A.H.] (*). **(42)** Although the day in Rajab was uncertain, he determined by mental calculation in which degree and minute and zodiacal sign the sun had been. When he dismounted, he asked me [about it] to put me to the test. I, too, by mental calculation, determined that it had been 23 degrees of **(43)** Virgo, but actually I could not determine the minutes accurately, because it was difficult by mental calculation and the fractions escaped [my] memory. However, it is certain that none of the astronomers whom I have met can carry out that mental calculation without a *zīj*. *Maulānā* ʿImād (*) cannot determine it [even] with a *zīj* if he does not have the calendar of that year; **(44)** even so, he has to check one by one the five doubtful days [in question].

[His Majesty] has solved [the problems of] the *Tuḥfa* (*), and has made valid objections to its author. I also studied every [faulty] issue that he had found out, and the facts and details of that [issue] were determined; [then] by discussing with the masters of this art, that issue was **(45)** made clear to all. His Majesty remembers all [the debated issues] after five or six months or [even] a year, whereas we [His Majesty's] servants only may remember the generalities, but certain details thereof escape us, and we need some study or reflection. [As for] Qāżīzāda, he needs to study [the cases again] even after three days or a week.

[...] there used to be in Herāt a **(46)** scholar named Yūsuf Ḥallāj (*). He had a son who had gone to Egypt to study, and had stayed there and [later] in Syria, Anatolia, and [adjacent] regions. He had applied himself with utmost diligence in this branch [of science], and had procured for himself a lot of books in this science. He has some knowledge in medicine, too. He had [attracted] many students. Having heard about the renown of the observatory, he came **(47)** to Samarkand, and started vaunting his erudition and his competence in mathematics. [Meanwhile] I had gone with the royal retinue to Bukhārā. [Reportedly] he had made a circular (*pargār*) calendar consisting of a bunch of paper, and he possessed several kinds of astrolabe which he showed to people [pretending] that nobody had ever seen anything like them. After our return from there, **(48)** one day His Majesty was at the observatory site, attended by Qāżīzāda and me. The story [of Yūsuf Ḥallāj's son] was brought up, and he was said to wish to attend [His Majesty]. [It was reported that] he had mad

a circular calendar, had strange astrolabes, including an amazing gadget called *zarqāla* (*) with an instruction manual to use it, which he obtained in Syria or Egypt, and which contains **(49)** forty-six chapters. [The day] when he [i.e., Yūsuf's son] came to kiss the [royal] carpet, he did not present [his] calendar, because during his stay in the city, he had heard about the situation, about my calendar in which I had calculated the orbits of stars in parasangs, and about the treatise I have written on the composition of antidotes to poisons. But he declared that he had a variety of astrolabes, and that he would present **(50)** intricate instruments. The day when his story was told at the site of the observatory, His Majesty asked Qāżīzāda, who had visited him: "What is it, that *zarqāla*?" He answered: "It is a plate on which are drawn many lines, some straight, some curved or circular. One cannot know **(51)** what it exactly is." I asked Qāżīzāda whether those lines were in the form of almucantars [i.e., parallels of altitude] or in another form. He replied: "No, they had a particular form. It is an odd gadget. It cannot be used without an [instruction] book." I said: "Why should be it impossible? Hasn't somebody invented it? We may figure it out **(52)** if we see [the instrument] with our own eyes." Anyhow, the other day when *Maulānā* Ḥallāj's son came to the [royal] presence, he had that instrument [i.e., the *zarqāla*] with him, which he handed over to His Majesty. His Majesty ordered me to go near him; he was holding it with his blessed hand. I looked at it. I figured out at once what it was and how the planispheric projection had been done, **(53)** and presented a brief survey [thereof]. Since His Majesty is very familiar with the science of planispheric projection, his blessed mind realized that I had solved [the problem]. [Then] I said: "Why [those] forty-six chapters? If you want me to, I will write a hundred chapters on how to know it [i.e., the *zarqāla*]." Thereupon I went home, drew a plate like that after the single look I had taken at it, and by the end of the day I **(54)** submitted it [to His Majesty]. I did not remember that the day when I had asked Qāżīzāda about it, I was at the observatory site and His Majesty was present there. So I told His Majesty that one day I had asked Qāżīzāda about the *zarqāla* in possession of Yūsuf Ḥallāj's son, and he had said it was a strange thing **(55)** utterly inexplicable. His Majesty said: "It was in my presence that you asked him about that, and he answered that he could not at all determine what it was." In short, although *Maulānā* Ḥallāj's son has a group [around him] and many students, when he entered such an arena where the others consider **(56)** themselves superior to him, and do not recognize him [as a scholar] just because he has been to Egypt and Syria, he abandoned his claim to this art, and did not present his calendar. Now he has an apothecary shop where he is engaged in a dull medical activity for the people in a bazaar near his home. The [champions] in the field keep repeating: "Surely the small bird does not become an eagle in our land, and to us distinction between silver and chalk is [evident]." (*)

(57) We come [now] to the subjects of prosody and [musical] scales (*adwār*) (*). Although the science of [musical] scales is [part] of mathematics, I had no knowledge

of it because until recently, I had not found a treatise about it. I wished to acquire some knowledge in it to complete [my knowledge of] the art [i.e., mathematics]; but being engaged in the work for the observatory, as **(58)** you have advised me, I would not occupy myself with anything else. I did not bend my mind to it until last year when we were honored by the visit of His Excellency *Khwāja* 'Abd al-Qādir (*). His Majesty who had previously studied the *Adwār* [i.e., the treatise on musical scales by Urmawī (*)], wanted to study it again. In the *Adwār* there is a difficult problem concerning the division of lute **(59)** frets, which [problem] is closely related to arithmetic. The earlier [scholars] have not been able to solve it properly. The author of the *Adwār* has tried his utmost [to explain it], and has found many subtle points, but [his solution] is not yet free from every flaw, and [the author] admits it [i.e., its deficiency]. *Maulānā* Quṭb al-Dīn Shīrāzī (*) wrote a lengthy treatise on [musical] scales, in which he duly glorified the author **(60)** of the *Adwār*, [but] did not deal at all with that intricate problem; and *Maulānā* Yaḥyā Kāshī literally translated (*) the *Adwār* into Persian without paying any attention to [the problem]. His Majesty, may God perpetuate his reign, asked me if I had pondered at all over that problem; I said I did not have any copy of the *Adwār*. He ordered a copy thereof to be brought from the [royal book] **(61)** depository; he offered it to me, saying: "Go deeply [into that problem]; maybe you can solve [it]." I took the copy home, and *Ḥaqq* [the "True One," i.e., God] knows best of all that I studied it from beginning to end in one day. Thanks to divine guidance, to the solicitude of His Majesty, who uttered the words "you can solve [it]," and by the blessing of the good wishes of that **(62)** master [i.e., you, my father] which are always with me, the whole [content of] the book, including [that] difficult problem together with many supplementary matters were accomplished. For example, I cut a piece of wood like a lute finger-board, and divided it properly according to [the rules of] arithmetic and geometry [...] which I had found for it pleased [His Majesty] extremely. When [His Majesty] showed it to *Khwāja* 'Abd al-Qādir, the latter praised it highly. He fixed silk [threads] **(63)** on it, and listened to it to check its accuracy. He said: "For this art perfect [command of] arithmetic is necessary." I had presented [the scales] in a table along with a description. He [*Khwāja*] told me to give him a copy of the table. I wrote one for him, too.

The author of the *Adwār* has mentioned twelve kinds of pentachord, and *Khwāja* 'Abd al-Qādir has found one more, thus he has mentioned **(64)** thirteen in his writings. I augmented them to nineteen (*), and submitted them to *Khwāja* 'Abd al-Qādir, who praised [my work] highly. This was the reason why I studied the *Adwār*.

[... **(65)** ... **(78)** ...] (*)

[Finally,] as to the few other bits of advice that you have given me, they **(79)** are perfectly right. I do not have the opportunity of [doing] anything else. I study mathematical books every day. Since my arrival here, I have gone five times through the whole art from beginning to end, down to the minutest details. I also have five or six incomplete works on my hands, e.g., *Miftāḥ al-ḥisāb* [Key to Arithmetic] and *Amthila-ye a'māl-e zīj* [Examples of the Procedures of the Astronomical Handbooks with Tables] (*) **(80)**. There is no leisure time [left for me] to finish them.

By God's will, generosity and favor, [finishing them] will be made easy. I do not dare to go into more tedious details. By the truth of the True One, may [your] high shadows be widespread!

[Your] humblest servant

Ghiyāth.

COMMENTARY

References are to the lines of the manuscript, indicated in the translation by boldface numbers in parentheses.

1: *Maulānā*, literally "our master," was used as an honorific title for outstanding sufis, scientists, and the like.

1: *Raṣad* means (astronomical) observation; here it is also used to mean "observatory."

2: As was (and still is) customary in Persian, when writing or talking to a superior, a writer (or speaker) does not refer to himself as *man* ("I") with a verb in the first person singular, but uses polite forms such as *īn banda* ("this slave/servant"), with the corresponding verb, if any, in the third person singular. For simplicity's sake, I have used "I" / "me" / "my" (as the case may be) throughout the translation.

3: The quotation is taken from the prophetic tradition (*ḥadīth*): "There will be no prophet after me ..." (see [5, 4:439; 24, 6:336]).

5: The author, when referring to the King, usually uses obsequious expressions such as *(bandagī-e) ḥażrat-e salṭanat panāhī*, lit., "(the servitude [i.e., the servants] in) the presence of [him who is] the refuge/sanctuary of sultanate," or just *bandagī-e ḥażrat* or *salṭanat panāhī* or *ḥażrat-e pādeshāhī*, lit., "the presence of the Kingdom [i.e., the King]." When subject of a sentence, these expressions take a verb in the third person plural. Further, he regularly repeats the parenthetical sentence *khallad-Allāhu mulkahu wasulṭānahu*, "may God perpetuate his reign and sovereignty," after a reference to the king. For the sake of brevity, throughout the translation we have used "His (Royal) Majesty"/"he"/"his"/"him" as the case may be.

5: On Ulugh Beg (1394–1449) see [10, 13:535–537; 16, 2:492–495].

5: *Kopakī* dinars were gold coins credited to, and named after, a certain Kopak Khan who was a Mongol ruler.

9–10: The problem is shown in Fig. 1 (not to scale), in which AB and θ are to be found. This problem is also mentioned in the second letter. There, al-Kāshī says that the height of the observer is 3.5 cubits [12, 196; 21, 97-98]. According to al-Kāshī in his *Sullam al-samā'* [11, 34-36], R, the radius of the Earth, is 1,272 parasangs and the radius of the sphere of fixed stars is $26{,}328R$, where each parasang equals 12,000 cubits. On the basis of these values one can show that $AB \approx 1{,}477R$, and $\theta \approx 2' 12''$. For the definition of the true horizon, see [3, 48].

11–12: This problem for the equal hours is also found in the second letter [12, 195; 21, 97]. Kennedy says in his commentary:

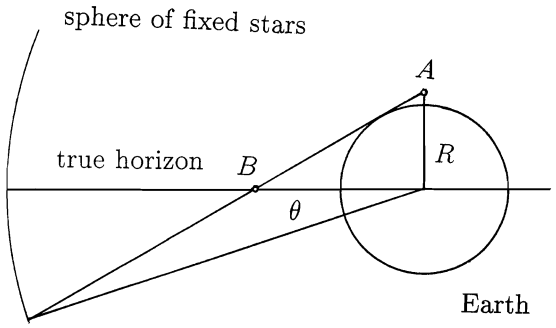

FIGURE 1

> The problem described here can be characterized as a ... problem in sundial theory.... In this case it is necessary to compute sets of horizontal coordinates of the sun corresponding to given times of the day and to given positions of the sun in the ecliptic. The latitude of the locality is of course a parameter of the function involved. For each determination the solar azimuth and altitude must then be laid out from the endpoint of the gnomon ... by means of a thread, or otherwise. The intersection of the thread with the wall gives one of the desired points, and the joining of proper sets of points by smooth curves completes the solution. Since Kāshī claims to have completed the job in one day it is probable that he used an astrolabe or some other such computing device to perform the requisite transformation of coordinates. [12, 208]

The problem for equal hours was solved by various medieval Islamic authors, such as Thābit ibn Qurra (see, for example, [9; 15]) and al-Marrākushī, who explained the practical computation of this type of sundial in Part 2, Book 3, Chapter 6 of his *Jāmi' al-mabādi' wa'l-ghāyāt fī 'ilm al-mīqāt* ("All beginnings and ends in the science of timekeeping"), see [23, 520-524, Fig. 102].

12: Ṣalāḥ al-Dīn Mūsā b. Muḥammad Qāżīzāda Rūmī [i.e., from Rūm = Anatolia], was born ca. 1365 in Bursa and died ca. 1435 in Samarkand. He was a Muslim mathematician and astronomer and a colleague of al-Kāshī in the Samarkand observatory. He joined Ulugh Beg's circle in Samarkand around 1410, and Ulugh Beg chose him as his mathematics teacher. It is likely that he introduced al-Kāshī to Ulugh Beg. He became the director of Ulugh Beg's school in 1420, where he also taught mathematics and astronomy. After al-Kāshī's death in 1429, he became the director of the Samarkand observatory. See [10, 11:227-229; 16, 2:487–489; 21, 39–43].

13–14: For this problem, which is also found in the second letter, Kennedy gives the following explanation:

> Abū Ḥanīfa prescribed that the time of the evening prayer begins at the instant when the length of the shadow cast by a vertical gnomon on a level surface equals twice the gnomon length. This implies that the angle of elevation of the sun above the western horizon shall be arc tan 1/2, or about 26.6°. Consider a point on ... a thick wall. The locus of those rays of the sun which pass through this point and satisfy the above condition is a right circular cone having its vertex at the point on the wall and its elements making an angle of 26.6° with the horizontal.

> If a narrow conical slot is made through the wall coinciding with part of the lateral surface of this cone, a ray of the sun will pass through the slot into the room at the beginning of the time for evening prayer, and only at this time, regardless of the season of the year. [12, 208]

In a recent personal communication, Professor Kennedy now writes: "The criterion that the shadow length shall be twice the gnomon length was in fact not given by Abū Ḥanīfa. This is a primitive rule which Abū Ḥanīfa modified. He prescribed instead that the time for afternoon prayer begins when the increasing length of the gnomon shadow equals the sum of twice the gnomon length plus the length of the noon shadow for that day. If this requirement is adopted, a solution to the problem proposed by Ulugh Beg is by no means simple. For the noon shadow at any latitude is a variable which depends on the season."

See also [2, 1:219].

16–18: This subject is also mentioned in the second letter [12, 197]. On al-Bīrūnī (973–1048) and his *Qānūn al-Mas'ūdī* see [10, 2:147–158; 16, 2:264–295].

19–22: Here the text is damaged and my restorations are conjectural. The problem is not mentioned in the second letter. I have not been able to identify the treatise which contains the chapter "On the curiosities of the astrolabe."

26–27: In this letter, al-Kāshī gives more detailed information on these instruments than in the second letter [12, 196; 21, 98–99]. The information on the Fakhrī sextant in Marāgha is puzzling because other sources indicate that there was no such instrument in the Marāgha observatory [12, 209; 21, 47; 22, 198–199]. Information on the Islamic observatories in the text can be found in [22]. Since al-Kāshī designed the Samarkand observatory, the precise information in (**26**) about the sextant puts an end to the old debate as to whether the stone arc of the observatory in Samarkand is a sextant or a quadrant.

27: *Maulānā* Muḥammad Khānī is probably the same as *Maulānā* Muḥammad 'Alīm (lit., "learned, erudite"), a famous scientist in Samarkand and a fellow student of Ulugh Beg who was very close to him. Because of his impudence, Ulugh Beg finally sent him to Herāt [17, 15, 192]. I have no further information on *Maulānā* Muḥammad Khānī.

28: 'Alī b. Muḥammad al-Jurjānī, called Sayyid Sharīf (1339/40-1413/4), was an Iranian scientist whom Timur sent from Shīrāz to Samarkand. He returned to Shīrāz after Timur's death. He wrote many scientific and philosophical works in Arabic and Persian, including a commentary on Chaghmīnī's *Mulakhkhaṣ fi'l-hay'a* (Compendium of Astronomy). See [6, 2:602–603; 16, 2:475–476].

28: *Maulānā* Abu'l-Fatḥ may be identified with Abu'l-Fatḥ Dhubāba (lit., "fly"), an abstemious man in the time of Ulugh Beg. He had special permission to meet the latter and to converse with him freely [13, 4:36].

36: The quotation is from the Koran 62:4 [14, 583].

36: The addition of "[not just]" makes the meaning clear and is made according to a similar phrase in the second letter [12, 193; 21, 94].

37: Auḥad al-Dīn Muḥammad (or 'Alī) b. Isḥāq, known as Anwarī Abīvardī (d. 1188/9 or 1190/1) was an Iranian poet from the city of Abīvard in northeastern

Iran. He was a prominent poet at the court of the Seljukid Sultan Sanjar, and noted for his command of Arabic literature, astronomy, and music [4, 2:364–391].

38: Ẓahīr al-Dīn Abu'l-Faẓl Ṭāhir b. Muḥammad, known as Ẓahīr-e Fāryābī (d. 1201/2), was an Iranian poet born in Fāryāb near the city of Balkh. He lived in Nīshābūr and Iṣfahān, and finally in Tabrīz where he died. Like Anwarī, he was famous for composing elegies (*qaṣīda*s) and for his skill in different sciences of his time [4, 2:412–425].

41: This year is given as 719 in the manuscript and 818 in the second letter. However, it seems that al-Kāshī intended the year 819 of the Islamic chronology, because in this year the sun was approximately in 24 degrees of Virgo on Monday Rajab 14, which fits in with the other data. This day coincides with September 7, 1416 A.D. In this year the vernal equinox was on March 12. Kennedy pointed out that the year 818 of the Hijra chronology cannot be the correct reading [12, 206].

43: *Maulānā* 'Imād is probably Mas'ūd b. Mu'tazz, also called 'Imād Niẓāmī Mashhadī (fl. ca. 1421), an Iranian mathematician who lived in Samarkand. See [16, 2:496; 19, 463].

44: For the *Tuḥfa* see note on **(59)** below and [16, 2:431].

46: I have no further information about Yūsuf Ḥallāj or his son.

48: The *zarqāla* was a universal astrolabe, invented by Abū Isḥāq Ibrāhīm b. Yaḥyā al-Naqqāsh, an Andalusian astronomer of the 11th century, known as al-Zarqāllu (or Arzakhel), see [10, 14:592–595; 16, 2:308–309]. There is now a considerable literature on the instrument and its history. For references see [20, 187–199].

56: The first part of the phrase quoted here is an old Arabic proverb changed here into negative form. In its original affirmative form, "the small bird becomes an eagle in our land," it is used to imply that anyone who comes to us will progress in his career.

57: The Arabic word *adwār* (plural of *dawr*) means "cycles" or "(musical) scales." On this discipline see [25].

58: 'Abd al-Qādir b. Ghaybī Marāghī (on whom see [6, 7:684, 1042]) was a famous Iranian musician who flourished in Bagdad. Timur took him to Samarkand, and he spent part of his life in Herāt where Shāhrokh (Ulugh Beg's father) ruled. *Khwāja* was a Persian honorific title which literally meant "lord" or "master."

58: *Adwār* is also the title of an important musical work written by Ṣafī al-Dīn Urmawī, a famous Iranian musician who lived in Bagdad in the 13th century (see [6, 8:805–807]).

59: Quṭb al-Dīn Shīrāzī (1236–1311) [10, 11:247–253; 16, 2:427–432] was an Iranian mathematician and astronomer. He was a disciple of Naṣīr al-Dīn Ṭūsī, and wrote *al-Tuḥfat al-shāhiyya fi'l hay'a* (The Royal Gift on Astronomy), sometimes briefly called *Tuḥfa*.

60: A manuscript of this Persian translation is extant in the Majlis library in Tehran, no. 2207. See [8, 6:168]. On the mathematical works of Yaḥyā b. Aḥmad al-Kāshī see [16, 2:446; 19, 322–323].

64: Al-Kāshī does not say anything precise about his additional pentachord types. From the 13th to the 15th century many scientists were interested in music theory,

and much experimenting and computing was done. The problem mentioned here is to find all possible divisions of the interval 3:2 into five notes so that the resulting scale will sound pleasant to the ear. In an Arabic commentary on the *Adwār* composed by a certain Mubārakshāh in 1375 (777 A.H.), seven additional pentachord types are mentioned [7, 3:344–350]. According to Dr. Eckhard Neubauer (Frankfurt), al-Kāshī probably had no direct knowledge of Mubārakshāh's commentary.

64–78: The lines dealing solely with prosody are omitted here, but the Persian text can be found in [1].

79: The *Miftāḥ al-ḥisāb* is extant and has been published; see [10, 7:261; 16, 2:481–483] for references. I know of no other reference to al-Kāshī's work *Amthila-ye a'māl-e zīj*; he may never have finished it.

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