A Newly Found Letter of Al-Kāshī on Scientific Life in Samarkand

Mohammad Bagheri
Encyclopaedia Islamica Foundation, P.O. Box 13145-1785, Teheran, Iran

The famous 15th-century Iranian mathematician and astronomer, Jamshid al-Kāshi, left his native Kāshān for Samarkand in order to participate in the scientific activity there, upon Ulugh Beg's invitation. Al-Kāshi corresponded in Persian with his father, who lived in Kāshān. One of his letters to his father was published and translated by Kennedy (1960) and Sayih (1960). In this paper, we present an English translation with commentary of another letter of al-Kāshi to his father, which has been found recently in Iran. Like the previous one, this new letter contains interesting information on Ulugh Beg's scientific circle in Samarkand.

چکیده
جمهشی کاشانی ریاضیدان و اخترشناس نامدار ایرانی در قرن نهم هجری، به دعوتalg بیگ برای شرکت در فعالیت علمی از شهر خود کاشان به سمرقند رفت. کاشانی باپدرش که در کاشان می‌زیست به فارسی مکتوبه‌ی می‌کرد. یکی از نامه‌های او بهپدرش آغاز شناخته شده و به زبان‌های مختلف ترجمه شده و منتشر شده است. در اینمقاله، نامه دیگری از جمهشی کاشانی که اخیراً یافته شده معرفی می‌شود. این نامهنویاسنده همانند نامه ای که قبلی شناخته شده بود حاوی اطلاعات جالبی درباره محفظعلمی alg بیگ در سمرقند است.

المتاص
انتقل غیبت‌الهای جمهشی کاشانی، ریاضیدان و اخترشناس نامدار ایرانی، از شهر خود کاشان به سمرقند. او به دعوت alg بیگ برای شرکت در فعالیت‌های علمی در سمرقند رفت. او برای پدرش که در کاشان زیست، نامه‌ای به زبان فارسی مکتوبه بود که در این مقاله به آن اشاره می‌شود.

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INTRODUCTION

Ghiyāth al-Dīn Jamshīd b. Mas'ūd b. Mahmūd Kāshānī, better known in Western literature as al-Kāshī, was a prominent Iranian mathematician and astronomer of the medieval Islamic world. He was born in Kāshān, where he was engaged in astronomical observations before his departure for Samarkand in 1421 (824 A.H.). He died in 1429 (832 A.H.) [10, 7:255–262; 16, 2:480–486; 18, 8].

Al-Kāshī joined the scientific circle of Ulugh Beg, the ruler of Samarkand, and was supported by him. He corresponded in Persian with his father, who lived in Kāshān and who apparently was familiar with mathematics and astronomy. One letter from al-Kāshī to his father was translated into English independently by Kennedy [12] and Sayyid [21] (with the Persian text). Russian, Arabic, Turkish, and Uzbek translations have appeared as well.

In this letter, al-Kāshī says that he repeats some stories about people’s opinion of him from an earlier letter which he had sent to his father “through the merchants of Qum” [12, 193; 21, 93]. In the winter of 1994, I found another letter from al-Kāshī to his father in the Majlis Library, Tehran, in MS. No. 5138/142. This manuscript is different from the letter published in [12;21], but al-Kāshī’s description of people’s opinion of him is similar to that in the published letter. Thus the manuscript must be a copy of the first letter, which was hitherto believed to be lost. From now on we will call the letter published in [12] and [21] the second letter. The first letter includes much new information, and even in the overlaps we find details which help clear up some phraseological or topical ambiguities in the second letter. The unique manuscript of the first letter is followed by a hitherto unknown manuscript of the second letter.

The first letter was written about two years after al-Kāshī’s arrival in Samarkand (see line 34 below), i.e., around A.D. 1423. The manuscript text of the letter has 80 lines. The numbers of the lines are indicated in boldface in the following English translation, for easy reference to the contents of the letter. The final section of the letter dealing solely with prosody (lines 64 to 79) is omitted here, because the contents are complicated and difficult to translate into English. This part and the illegible words in line 62 are shown as dots in brackets. The same notation is used for the location of an apparent lacuna in line 45. My own explanatory additions to the text appear in square brackets. An asterisk (*) in the translation refers to the commentary.

The manuscript is not an autograph, and some errors in the text seem to be due to the scribe’s carelessness. The proximity of the composition, especially in the preamble of the letter, has been avoided in the translation as far as possible. Where the exact meaning of a phrase was not clear to me, I have chosen the most probable one. The complete Persian text of the first letter with commentary, including the section on prosody, is published in [1].

I am glad to publish this extremely valuable document which—like the second letter—provides historians of science with a first-hand source about the scientific atmosphere of Samarkand in Ulugh Beg’s time.
TRADUCTION DE LA PREMIÈRE LETTRE \textsc{d'Al-Kashi à son père}

(1) [Suivait] le texte de la lettre de notre maître, le plus grand orgueil des philosophes [ou sages, \textit{hukama'}] et des géomètres, \textit{Maulānā} (*), Ghiyāth al-Dīn Jamshīd al-Kāshānī, rédigée à Kashan depuis l'ancienne capitale Samarkand alors qu'il était engagé dans les observations astronomiques (\textit{raṣād}) (*) là.


La situation dans la ville de Samarkand, que Dieu la protège de [l'épreuve], est [comme suit]. Son Excellence (5) [c'est-à-dire Ulugh Beg (*)] a donné [une] donation [caritative] de trente milliers de 

Well, when (8) I arrived here, as it is customary [everywhere], all of them began inquiring and spying in order to find out the extent of my knowledge [lit., “beginning”] in this art, so that, if they found me to have little knowledge or to be equal or slightly superior to them, they keep their positions; thus they might not [have to] say that a stranger came in and got the better of them. During the period that the arena was theirs, (9) in the discussions held in the presence of His Royal Majesty they were confronted with some difficulties into which they had looked for a month or two or even for a year, but to which no solution had been found. For example, this problem: [Let us suppose] somebody is standing on a perfectly circular ground or on the sea surface, and the visual ray issuing from his eyes is tangent (10) to that, and [then] reaches the sphere of the ecliptic [falak al-buri'ij, i.e. the outermost sphere of the universe]. Now, at which distance will [that ray] intersect the true horizon, and, where it reaches the sphere of the ecliptic, how much will it be depressed from the true horizon? (*) And many other [problems] which were detailed [to you] previously.

[Well,] the very day I reached Samarkand, although my baggage had not arrived yet and I had no book or zīj [set of astronomical tables] with me, (11) I borrowed a reed pen and ink-pot from a science student in a school, solved all [those problems], and submitted [my solutions] to His August Majesty. [Later,] His Majesty put me to the test in similar cases. [For instance,] I was ordered to set up a gnomon [miqāṣ] on a wall of the royal palace, the surface of which [wall] is not in alignment either with the meridian plane or with the initial plane of azimuths [i.e., the East-West plane], and to draw the lines of equal [i.e., equinoctial] and (12) unequal [i.e., seasonal] hours on it, that is to say, a sundial [rukhāmā] on the surface of a wall of unknown [i.e., arbitrary] azimuth. I drew them (*). All the specialists in [this] art came to view [the sundial], [including] Qāẓīzāda Rūmī (*), who is the most learned of all, and who confessed that this [scheme] was extremely difficult. He tried in vain to figure out the proof thereof, and finally asked me about it, (13) [thus] acknowledging my victory in the presence of many people. After examining [the sundial] in different ways, he avowed that [the scheme] was really well thought out. Similarly, His Majesty [once] said: “We would like to make a hole in the wall of a miḥrāb [prayer niche in a mosque] in such a way that the sun may shine through that hole for a short while at the afternoon [prayer] time both in summer and in winter. That single hole (14) must be round from inside, but from the outside it must be in such a way that sunshine cannot pass through it at times other than the afternoon [prayer time]. This [royal wish] had been [already] expressed before my arrival, and nobody had been able to realize it; [but] when I came [here], I did this also. (*)

Further, His Royal Majesty visits the school once every few days, and (15) attends a class for some time, and I am in His Majesty’s company. [On these occasions] we do not know at all what the lesson will be about, nor what problem will be brought up, [whereas] the teacher and students have studied it together the night before. I have [sometimes] intervened without preparation in a [discussion] and I have spoken so much about the point [in question] that everyone was astonished.
One day, His Majesty was reading a treatise in which a reference was made to *al-Qānūn al-Mas'ūdī* ("The Mas'ūdic Canon"), a work composed by Abū Rayhān al-Bīrūnī (*). He sent for the *Qānūn*, in which he found the problem; but because of some difficulty [in the problem], it was not understood. I happened to have quotidian fever for two days, and did not present (17) myself [at the court]. Qāżīzāda had been there. [His Majesty] had placed the *Qānūn* manuscript before him. [The problem] could not be solved then and there; [so] Qāţīzāda had taken [the manuscript] to his *wuthdq* [cell or chamber in a madrasa, i.e., school], and had meditated upon [the problem], but in vain. On the third day he had taken [the manuscript] back [to His Majesty], saying: "Certainly there is at this point a lacuna left by the scribe, for the problem is not fully solvable." At this point I arrived there. His Majesty, may God perpetuate his reign and sovereignty, pointed to me and (18) said: "There is a difficulty at this point. Think it over." And he passed the manuscript on to me with his blessed hand. I reflected awhile on the manuscript, found out [the solution], and submitted it immediately to His Majesty, who repeatedly expressed his admiration for me.

Another day, when arriving at the school, His Majesty had met a student at the door, (19) holding a book. He [i.e., the King] had asked him what book it was. [The student,] kissing the book, had presented it [to His Majesty]. Opening the book, [His Majesty] had chanced on a chapter [entitled] "On the curiosities of the astrolabe," [beginning with this problem]: "[Let us suppose that] the Sun is, e.g., in 10 degrees of Aquarius, with a certain altitude, and the ascendent of time is a certain degree [of the ecliptic]; then [the ascendent of the time when] its [i.e., the Sun's] altitude [is the maximum altitude of the ecliptic at that moment] is a quadrant [in advance of the Sun's position], i.e., in 10 degrees (20) of Taurus. After one month, the Sun having described one sign [of the Zodiac], while having the same altitude as on that [previous] day, how could the ascendent be exactly the same as it was in that day?" (*) After having entered [the classroom, His Majesty] had presented that problem for discussion. Someone said that this was most likely where the latitude of the city exceeds the [complement of the (?)] [sun's] declinations [i.e., plus or minus 67.5 degrees], and the signs rise inversely. They had (21) thought about it for a long time. I arrived there towards the end of the meeting. [His Majesty] immediately ordered the book to be handed to me, saying: "Figure out where this [case] could be." As soon as I had a look [at the problem], I said it could be at Samarkand, and I gave reasons [for my statement]. Some students did not (22) understand. [So] an astrolabe was brought, and I made [my point] clear to them. [Well,] what I mean is that [His Majesty] has recognized my competence after witnessing all that and many similar cases rather than by conjecture or on the authority of others.

Concerning the observatory, at first [His Majesty] had imagined that Ptolemy's method should be faithfully followed. [Consequently] he had ordered two rings [to be made], each 6 (23) cubits (gaz) in diameter and 2 digits (isba') in thickness. He was unaware [of the fact] that later [scholars] had discovered [new] subtle points [i.e., improvements] and had made more precise [instruments]. [For instance,] he
certainly did not know about the quality of the Marāgha observatory. [When] I came, I told [him] that in the observatory constructed in 'Azud al-Daula's time the ring made for it was 10 cubits in diameter and one span of the hand in thickness so that the ring might not be distorted; and that, however, subsequent astronomers had turned away from it because of some defects inherent in it, and had relied on the Fakhrī sextant invented in Fakhr al-Daula's time. I explained the cause of those defects, and [I added that] the same Fakhrī sextant had been constructed at the Marāgha observatory. (25) Because His Majesty had seen the internal parts of the latter in his childhood, as soon as I pointed out those parts to him, he remembered them, and said that I was right and that was so. [Thus] he made up his mind [that] the rings which had been cast were decidedly of no use, and that they were not even worth being completed as armillary spheres, for they were too thin. [So] those rings were [ordered to be] broken, (26) and an observatory like that of Marāgha was founded as I had suggested. However, an innovation has been made in the position of the Fakhrī sextant: The building was designed in circular form with a perimeter of 200 cubits of Kāshān; it stands on top of a rock, in which part of the sextant was carved, so that the edifice might not be very tall, because, bricks not being firm enough here, (27) excessive height of a building may cause fracture. At Marāgha, the sextant is positioned higher [but] the other [compartments of the observatory] are not so high, [which is] a bad configuration. Here, the surface of the roof, too, will be flat, so that other astronomical instruments may be placed on it. (*)

Moreover, there are three teachers in the school of His Majesty, may God perpetuate his reign and sovereignty: (1) Qāzīzāda, (2) Maulānā Muhammad Khānī (*), [who] (28) surpasses the others by far in sciences other than mathematics; he is endowed with a phenomenal memory, so he teaches from memory most science lessons, and he studied astronomy with Sayyid Sharīf (*); (3) Maulānā Abu'l-Fath (*), whom His Majesty met in Herāt, who is versed in Islamic jurisprudence, and has composed [a tract on] astrolabe reading. He teaches only the latter subject. In short, Maulānā Muhammad Khānī, (29) when occasionally being present [at the court] or meeting [His Majesty] at the observatory site, has repeatedly said in all fairness: ‘I used to be singled out in Samarkand for my knowledge of astronomy, and nobody could compete with me. Now, Qāzīzāda, having practiced a lot, has surpassed me, and Maulānā Ghiyāth al-Din accomplishes miracles of presence of mind. To do them justice, [I must admit that] (30) when there is a discussion in their meeting, I dare not intervene, all the more so because His Majesty knows this art well and [therefore] one cannot impudently claim competence. The subject of the problem [to be submitted in the session] is already known [to the students; therefore], presence of mind is a must, and we [i.e., I] happen to lack presence of mind in this art. All night long we study a lesson and the next morning we teach it with a thousand artifices. But there is no need for extensive studying in other sciences.’

(31) By annoying you [with these details], I intend [to let you know] that I have entered a field where my colleagues in this art are Avicennas [i.e., top experts] in
other sciences. [However,] on account of [my] presence of mind and of my investigations in all branches of this art, I have come to a point where His Majesty the World-Conqueror, may God perpetuate his reign and sovereignty until the Day of Resurrection, has repeatedly said in assemblies and (32) gatherings: "In this art Maulānā Ghiyāth al-Dīn's presence of mind is greater than Qāızīzāda's, and his knowledge and intellect are superior to the latter's, so much so that Maulānā Ghiyāth al-Dīn solves immediately any difficult problem that comes up, [whereas] Qāızīzāda thinks it over for several days and fails (33) to solve it." Similarly, I have adopted here such a way of life that His Majesty, may God perpetuate his reign and sovereignty, has said: "He is really a nice man, and has a decent mode of life. No matter which one of the mulūs [i.e., learned men] who came to us, for example, such and such a person, after we gave him sympathetic consideration for a week, he became so conceited that he began (34) quarreling with people and committing indiscretions. Maulānā Ghiyāth al-Dīn has been here for two years now. We have given him all manners of regard and solicitude; we converse with him every day, and we have often expressed our admiration for him. Anyone else would have started indiscretions. During all this time he certainly has not quarreled with anybody or (35) complained about anybody, and no one has ever complained about him. He did not tell on anybody in our presence out of ambition and covetousness. He does not mix with anybody; he is always busy with [his] work. He is a nice fellow." I thank the exalted and sanctified Creator for these two [royal considerations], because people constantly endeavor to go about both the acquisition of arts and [adoption of] a mode of life in such a way that they may be admired by their neighbors (36) and colleagues of the same class. By the grace and success granted by Almighty God, I behaved in a way that such a learned and talented king admires [me] in both regards. "That is the bounty of God; [He gives it to] whom He will." (*) As for one's livelihood, it is [generally] admitted that nobody knows about it better than kings.

[Now] I will mention the King's skills [not just] (*) by way of courtesy. First, he knows Arabic syntax well, (37) and writes [Arabic] elegantly. He knows Arabic, Persian, Turkish, Mongolian and some Chinese. He knows by heart all the glorious Koran, and knows well the commentary and citations thereof, thereby making apt quotations [therefrom] when speaking. He has memorized all the dates [for example, of events which occurred during his life]. He knows the art of prosody and versification extremely well. He knows by heart Anwāri's (*) whole divan and some poems (38) by Žāhir Fāyābī (*). He occasionally composes excellent poems after Anwāri. I do not remember anything [of his poems] to write down [here].

His fantastic memory is one of the wonders of the world. For instance, one day he sat down and began recounting all the places and dates of his residence since he had been in Sultāniyya, telling, [for example,] what day (39) of the week and the month it was when he [and his party] left such and such locality, halted for noon prayers at such and such place, and leaving there, when and where they traveled by night, and where they halted next. He [thus] came to the point of narrating what the envoys and spies had reported [to him during the trek]. He had
the diaries [of that long journey] brought to him [and read out]. We were listening. [The diaries] accorded [with his narrative]. [Subsequent events] down to that day when we were at that (40) meeting [had also been recorded]. Those [details] which had been recorded accorded with what he had [just] said. I had recorded some of those days in [my] diary—for example, the dates of my reaching Herat, departing [thence] for Samarkand, and moving from garden to garden in Samarkand. When he came down to these [events], I took up [my] diary; on the whole, [what he related] was in accordance [with my records] (41) as to the day of the week and the month, and he recounted most of the things that had happened on a [particular] day.

He has great skill in mathematics. For instance, once when he was out hunting, while on horseback he wanted to determine [exactly] when in summer or winter had occurred an event which was known to have taken place on a Monday between the 10th and 15th of Rajab 819 [A.H.] (*). (42) Although the day in Rajab was uncertain, he determined by mental calculation in which degree and minute and zodiacal sign the sun had been. When he dismounted, he asked me [about it] to put me to the test. I, too, by mental calculation, determined that it had been 23 degrees of (43) Virgo, but actually I could not determine the minutes accurately, because it was difficult by mental calculation and the fractions escaped [my] memory. However, it is certain that none of the astronomers whom I have met can carry out that mental calculation without a zīj, Maulānā ‘Imād (*) cannot determine it [even] with a zīj if he does not have the calendar of that year; (44) even so, he has to check one by one the five doubtful days [in question].

[His Majesty] has solved [the problems of] the Tuhfā (*), and has made valid objections to its author. I also studied every [faulty] issue that he had found out, and the facts and details of that [issue] were determined; [then] by discussing with the masters of this art, that issue was (45) made clear to all. His Majesty remembers all [the debated issues] after five or six months or [even] a year, whereas we [His Majesty's] servants only may remember the generalities, but certain details thereof escape us, and we need some study or reflection. [As for] Qāẓīzāda, he needs to study [the cases again] even after three days or a week.

[..] there used to be in Herat a (46) scholar named Yūsuf Ḥallāj (*). He had a son who had gone to Egypt to study, and had stayed there and [later] in Syria, Anatolia, and [adjacent] regions. He had applied himself with utmost diligence in this branch [of science], and had procured for himself a lot of books in this science. He has some knowledge in medicine, too. He had [attracted] many students. Having heard about the renown of the observatory, he came (47) to Samarkand, and started vaunting his erudition and his competence in mathematics. [Meanwhile] I had gone with the royal retinue to Bukhārā. [Reportedly] he had made a circular (pargār) calendar consisting of a bunch of paper, and he possessed several kinds of astrolabe which he showed to people [pretending] that nobody had ever seen anything like them. After our return from there, (48) one day His Majesty was at the observatory site, attended b Qāẓīzāda and me. The story [of Yūsuf Ḥallāj’s son] was brought up, and he was said to wish to attend [His Majesty]. [It was reported that] he had mad
a circular calendar, had strange astrolabes, including an amazing gadget called zarqāla (*) with an instruction manual to use it, which he obtained in Syria or Egypt, and which contains (49) forty-six chapters. [The day] when he [i.e., Yūṣuf's son] came to kiss the [royal] carpet, he did not present [his] calendar, because during his stay in the city, he had heard about the situation, about my calendar in which I had calculated the orbits of stars in parasangs, and about the treatise I have written on the composition of antidotes to poisons. But he declared that he had a variety of astrolabes, and that he would present (50) intricate instruments. The day when his story was told at the site of the observatory, His Majesty asked Qāızīzāda, who had visited him: “What is it, that zarqāla?” He answered: “It is a plate on which are drawn many lines, some straight, some curved or circular. One cannot know (51) what it exactly is.” I asked Qāızīzāda whether those lines were in the form of almucantars [i.e., parallels of altitude] or in another form. He replied: “No, they had a particular form. It is an odd gadget. It cannot be used without an [instruction] book.” I said: “Why should be it impossible? Hasn’t somebody invented it? We may figure it out (52) if we see [the instrument] with our own eyes.” Anyhow, the other day when Maulānā Ḥallāj’s son came to the [royal] presence, he had that instrument [i.e., the zarqāla] with him, which he handed over to His Majesty. His Majesty ordered me to go near him; he was holding it with his blessed hand. I looked at it. I figured out at once what it was and how the planispheric projection had been done, (53) and presented a brief survey [thereof]. Since His Majesty is very familiar with the science of planispheric projection, his blessed mind realized that I had solved [the problem]. [Then] I said: “Why [those] forty-six chapters? If you want me to, I will write a hundred chapters on how to know it [i.e., the zarqāla].” Thereupon I went home, drew a plate like that after the single look I had taken at it, and by the end of the day I (54) submitted it [to His Majesty]. I did not remember that the day when I had asked Qāızīzāda about it, I was at the observatory site and His Majesty was present there. So I told His Majesty that one day I had asked Qāızīzāda about the zarqāla in possession of Yūṣuf Ḥallāj’s son, and he had said it was a strange thing (55) utterly inexplicable. His Majesty said: “It was in my presence that you asked him about that, and he answered that he could not at all determine what it was.” In short, although Maulānā Ḥallāj’s son has a group [around him] and many students, when he entered such an arena where the others consider (56) themselves superior to him, and do not recognize him [as a scholar] just because he has been to Egypt and Syria, he abandoned his claim to this art, and did not present his calendar. Now he has an apothecary shop where he is engaged in a dull medical activity for the people in a bazaar near his home. The [champions] in the field keep repeating: “Surely the small bird does not become an eagle in our land, and to us distinction between silver and chalk is [evident].” (*)

(57) We come [now] to the subjects of prosody and [musical] scales (adwār) (*). Although the science of [musical] scales is [part] of mathematics, I had no knowledge

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of it because until recently, I had not found a treatise about it. I wished to acquire some knowledge in it to complete [my knowledge of] the art [i.e., mathematics]; but being engaged in the work for the observatory, as (58) you have advised me, I would not occupy myself with anything else. I did not bend my mind to it until last year when we were honored by the visit of His Excellency Khwāja ʿAbd al-Qādīr (*). His Majesty who had previously studied the Adwār [i.e., the treatise on musical scales by Urmawi (*)], wanted to study it again. In the Adwār there is a difficult problem concerning the division of lute (59) frets, which [problem] is closely related to arithmetic. The earlier [scholars] have not been able to solve it properly. The author of the Adwār has tried his utmost [to explain it], and has found many subtle points, but [his solution] is not yet free from every flaw, and [the author] admits it [i.e., its deficiency]. Maulānā Qutb al-Dīn Shirāzī (*) wrote a lengthy treatise on [musical] scales, in which he duly glorified the author (60) of the Adwār, [but] did not deal at all with that intricate problem; and Maulānā Yahyā Kāshī literally translated (*) the Adwār into Persian without paying any attention to [the problem]. His Majesty, may God perpetuate his reign, asked me if I had pondered at all over that problem: I said I did not have any copy of the Adwār. He ordered a copy thereof to be brought from the [royal book] (61) depository; he offered it to me, saying: “Go deeply [into that problem]; maybe you can solve [it].” I took the copy home, and Haqq [the “True One,” i.e., God] knows best of all that I studied it from beginning to end in one day. Thanks to divine guidance, to the solicitude of His Majesty, who uttered the words “you can solve [it],” and by the blessing of the good wishes of that (62) master [i.e., you, my father] which are always with me, the whole [content of] the book, including [that] difficult problem together with many supplementary matters were accomplished. For example, I cut a piece of wood like a lute finger-board, and divided it properly according to [the rules of] arithmetic and geometry […] which I had found for it pleased [His Majesty] extremely. When [His Majesty] showed it to Khwāja ʿAbd al-Qādīr, the latter praised it highly. He fixed silk [threads] (63) on it, and listened to it to check its accuracy. He said: “For this art perfect [command of] arithmetic is necessary.” I [Khwaja] told me to give him a copy of the table. I wrote one for him, too.

The author of the Adwār has mentioned twelve kinds of pentachord, and Khwāja ʿAbd al-Qādīr has found one more, thus he has mentioned (64) thirteen in his writings. I augmented them to nineteen (*), and submitted them to Khwāja ʿAbd al-Qādīr, who praised [my work] highly. This was the reason why I studied the Adwār.

[ … (65) … (78) … ] (*)

[Finally,] as to the few other bits of advice that you have given me, they (79) are perfectly right. I do not have the opportunity of [doing] anything else. I study mathematical books every day. Since my arrival here, I have gone five times through the whole art from beginning to end, down to the minutest details. I also have five or six incomplete works on my hands, e.g., Mīṣāḥ al-hisāb [Key to Arithmetic] and Amthila-ye aʾmāl-e zīj [Examples of the Procedures of the Astronomical Handbooks with Tables] (*) (80). There is no leisure time [left for me] to finish them.
By God's will, generosity and favor, [finishing them] will be made easy. I do not dare to go into more tedious details. By the truth of the True One, may [your] high shadows be widespread!

[Your] humblest servant

Ghiyath.

COMMENTARY

References are to the lines of the manuscript, indicated in the translation by boldface numbers in parentheses.

1: *Maulānā*, literally “our master,” was used as an honorific title for outstanding sufis, scientists, and the like.

1: *Rasad* means (astronomical) observation; here it is also used to mean “observatory.”

2: As was (and still is) customary in Persian, when writing or talking to a superior, a writer (or speaker) does not refer to himself as *man* (“I”) with a verb in the first person singular, but uses polite forms such as *in banda* (“this slave/servant”), with the corresponding verb, if any, in the third person singular. For simplicity’s sake, I have used “I” / “me” / “my” (as the case may be) throughout the translation.

3: The quotation is taken from the prophetic tradition (*ḥadīth*): “There will be no prophet after me ...” (see [5, 4:439; 24, 6:336]).

5: The author, when referring to the King, usually uses obsequious expressions such as *(bandagi-e) hażrat-e saltanat panāhī*, lit., “(the servitude [i.e., the servants] in) the presence of [him who is] the refuge/sanctuary of sultanate,” or just *bandagi-e hażrat* or *saltanat panāhī* or *hażrat-e pādeshāhī*, lit., “the presence of the Kingdom [i.e., the King].” When subject of a sentence, these expressions take a verb in the third person plural. Further, he regularly repeats the parenthetical sentence *khallad-Allāhu mulkahu wasutānahu*, “may God perpetuate his reign and sovereignty,” after a reference to the king. For the sake of brevity, throughout the translation we have used “His (Royal) Majesty” /“he” /“his” /“him” as the case may be.


5: *Kopakī* dinars were gold coins credited to, and named after, a certain Kopak Khan who was a Mongol ruler.

9–10: The problem is shown in Fig. 1 (not to scale), in which $AB$ and $\theta$ are to be found. This problem is also mentioned in the second letter. There, al-Kāshi says that the height of the observer is 3.5 cubits [12, 196; 21, 97–98]. According to al-Kāshi in his *Sullam al-samā‘* [11, 34–36], $R$, the radius of the Earth, is 1,272 parasangs and the radius of the sphere of fixed stars is 26,328$R$, where each parasang equals 12,000 cubits. On the basis of these values one can show that $AB \approx 1,477R$, and $\theta \approx 2°\,12'$. For the definition of the true horizon, see [3, 48].

11–12: This problem for the equal hours is also found in the second letter [12, 195; 21, 97]. Kennedy says in his commentary:
The problem described here can be characterized as a problem in sundial theory. In this case it is necessary to compute sets of horizontal coordinates of the sun corresponding to given times of the day and to given positions of the sun in the ecliptic. The latitude of the locality is of course a parameter of the function involved. For each determination the solar azimuth and altitude must then be laid out from the endpoint of the gnomon by means of a thread, or otherwise. The intersection of the thread with the wall gives one of the desired points, and the joining of proper sets of points by smooth curves completes the solution. Since Kashi claims to have completed the job in one day it is probable that he used an astrolabe or some other such computing device to perform the requisite transformation of coordinates. [12, 208]

The problem for equal hours was solved by various medieval Islamic authors, such as Thābit ibn Qurra (see, for example, [9; 15]) and al-Marrākushi, who explained the practical computation of this type of sundial in Part 2, Book 3, Chapter 6 of his Jam' al-mabādi' wa'l-ghāyāt fī 'ilm al-miqāt (“All beginnings and ends in the science of timekeeping”), see [23, 520-524, Fig. 102].

12: Ṣalāḥ al-Dīn Mūsā b. Muḥammad Qāzīzāda Rūmī [i.e., from Rūm = Anatolia], was born ca. 1365 in Bursa and died ca. 1435 in Samarkand. He was a Muslim mathematician and astronomer and a colleague of al-Kāshi in the Samarkand observatory. He joined Ulugh Beg’s circle in Samarkand around 1410, and Ulugh Beg chose him as his mathematics teacher. It is likely that he introduced al-Kāshi to Ulugh Beg. He became the director of Ulugh Beg’s school in 1420, where he also taught mathematics and astronomy. After al-Kāshi’s death in 1429, he became the director of the Samarkand observatory. See [10, 11:227-229; 16, 2:487–489; 21, 39–43].

13–14: For this problem, which is also found in the second letter, Kennedy gives the following explanation:

Abū Ḥanīfah prescribed that the time of the evening prayer begins at the instant when the length of the shadow cast by a vertical gnomon on a level surface equals twice the gnomon length. This implies that the angle of elevation of the sun above the western horizon shall be arc tan 1/2, or about 26.6°. Consider a point on a thick wall. The locus of those rays of the sun which pass through this point and satisfy the above condition is a right circular cone having its vertex at the point on the wall and its elements making an angle of 26.6° with the horizontal.
If a narrow conical slot is made through the wall coinciding with part of the lateral surface of
this cone, a ray of the sun will pass through the slot into the room at the beginning of the
time for evening prayer, and only at this time, regardless of the season of the year. [12, 208]

In a recent personal communication, Professor Kennedy now writes: “The criterion that the shadow length shall be twice the gnomon length was in fact not given
by Abū Ḥanīfa. This is a primitive rule which Abū Ḥanīfa modified. He prescribed
instead that the time for afternoon prayer begins when the increasing length of the
gnomon shadow equals the sum of twice the gnomon length plus the length of the
noon shadow for that day. If this requirement is adopted, a solution to the problem
proposed by Ulugh Beg is by no means simple. For the noon shadow at any latitude
is a variable which depends on the season.”

See also [2, 1:219].

16–18: This subject is also mentioned in the second letter [12, 197]. On al-Bīrūnī
(973–1048) and his Qānūn al-Masʿūdī see [10, 2:147–158; 16, 2:264–295].

19–22: Here the text is damaged and my restorations are conjectural. The problem
is not mentioned in the second letter. I have not been able to identify the treatise which contains the chapter “On the curiosities of the astrolabe.”

26–27: In this letter, al-Kāshī gives more detailed information on these instruments than in the second letter [12, 196; 21, 98–99]. The information on the Fakhri
sextant in Marāqīh is puzzling because other sources indicate that there was no such
instrument in the Marāqīh observatory [12, 209; 21, 47; 22, 198–199]. Information on
the Islamic observatories in the text can be found in [22]. Since al-Kāshī designed
the Samarkand observatory, the precise information in (26) about the sextant puts
an end to the old debate as to whether the stone arc of the observatory in Samarkand
is a sextant or a quadrant.

27: Maulānā Muḥammad Khānī is probably the same as Maulānā Muḥammad
‘Alīn (lit., “learned, erudite”), a famous scientist in Samarkand and a fellow student
of Ulugh Beg who was very close to him. Because of his impudence, Ulugh Beg
finally sent him to Herāt [17, 15, 192]. I have no further information on Maulānā
Muḥammad Khānī.

28: ‘Alī b. Muḥammad al-Jurjānī, called Sayyid Sharīf (1339/40–1413/4), was an
Iranian scientist whom Timur sent from Shirāz to Samarkand. He returned to Shirāz
after Timur’s death. He wrote many scientific and philosophical works in Arabic
and Persian, including a commentary on Chaghmīnī’s Mulakhkhāṣṣ fiʾl-hay’a (Compendium of Astronomy). See [6, 2:602–603; 16, 2:475–476].

28: Maulānā Abu’l-Fath may be identified with Abu’l-Fath Dhubābā (lit., “fly”),
an abstemious man in the time of Ulugh Beg. He had special permission to meet
the latter and to converse with him freely [13, 4:36].

36: The quotation is from the Koran 62:4 [14, 583].

36: The addition of “[not just]” makes the meaning clear and is made according
to a similar phrase in the second letter [12, 193; 21, 94].

37: Auḥad al-Dīn Muḥammad (or ‘Alī) b. Iṣḥāq, known as Anwārī Abīvardī (d.
1188/9 or 1190/1) was an Iranian poet from the city of Abīvard in northeastern
Iran. He was a prominent poet at the court of the Seljukid Sultan Sanjar, and noted for his command of Arabic literature, astronomy, and music [4, 2:364–391].

38: Zahir al-Din Abü’l-Faţl Tahir b. Muḥammad, known as Zahir-e Fārīyābī (d. 1201/2), was an Iranian poet born in Fārīyāb near the city of Balkh. He lived in Nishābūr and Iṣfahān, and finally in Tabriz where he died. Like Anwārī, he was famous for composing elegies (qaṣīdas) and for his skill in different sciences of his time [4, 2:412–425].

41: This year is given as 719 in the manuscript and 818 in the second letter. However, it seems that al-Kāshi intended the year 819 of the Islamic chronology, because in this year the sun was approximately in 24 degrees of Virgo on Monday Rajab 14, which fits in with the other data. This day coincides with September 7, 1416 A.D. In this year the vernal equinox was on March 12. Kennedy pointed out that the year 818 of the Hijra chronology cannot be the correct reading [12, 206].

Maulānā ‘Imād is probably Maṣʿūd b. Muṭazz, also called ‘Imād Nizāmī Mashhādī (fl. ca. 1421), an Iranian mathematician who lived in Samarkand. See [16, 2:496; 19, 463].

44: For the Tuhfā see note on (59) below and [16, 2:431].

46: I have no further information about Yūsuf Ḥallaj or his son.

48: The zarrqālā was a universal astrolabe, invented by Abū Ishaq Ibrāhīm b. Yahyā al-Naqqāsh, an Andalusi astronomer of the 11th century, known as al-Zarqālū (or Arzakhel), see [10, 14:592–595; 16, 2:308–309]. There is now a considerable literature on the instrument and its history. For references see [20, 187–199].

56: The first part of the phrase quoted here is an old Arabic proverb changed here into negative form. In its original affirmative form, “the small bird becomes an eagle in our land,” it is used to imply that anyone who comes to us will progress in his career.

57: The Arabic word adwār (plural of dawr) means “cycles” or “(musical) scales.” On this discipline see [25].

58: ‘Abd al-Qādir b. Ghaybī Marāghī (on whom see [6, 7:684, 1042]) was a famous Iranian musician who flourished in Bagdad. Timur took him to Samarkand, and he spent part of his life in Herāt where Shāhrokh (Ulugh Beg’s father) ruled. Khwāja was a Persian honorific title which literally meant “lord” or “master.”

58: Adwār is also the title of an important musical work written by Šafī al-Dīn Urmawī, a famous Iranian musician who lived in Bagdad in the 13th century (see [6, 8:805–807]).

59: Outb al-Dīn Shirāzī (1236–1311) [10, 11:247–253; 16, 2:427–432] was an Iranian mathematician and astronomer. He was a disciple of Naṣīr al-Dīn Tūsī, and wrote al-Tuhfat al-shāhiyya fi’l hay’ā (The Royal Gift on Astronomy), sometimes briefly called Tuhfā.

60: A manuscript of this Persian translation is extant in the Majlis library in Tehran, no. 2207. See [8, 6:168]. On the mathematical works of Yahyā b. Aḥmad al-Kāshī see [16, 2:446; 19, 322–323].

64: Al-Kāshī does not say anything precise about his additional pentachord types. From the 13th to the 15th century many scientists were interested in music theory,
and much experimenting and computing was done. The problem mentioned here is to find all possible divisions of the interval 3:2 into five notes so that the resulting scale will sound pleasant to the ear. In an Arabic commentary on the Adwār composed by a certain Mubārakshāh in 1375 (777 A.H.), seven additional pentachord types are mentioned [7, 3:344–350]. According to Dr. Eckhard Neubauer (Frankfurt), al-Kāshi probably had no direct knowledge of Mubārakshāh’s commentary.

64–78: The lines dealing solely with prosody are omitted here, but the Persian text can be found in [1].

79: The Miftalā‘ al-hisbāb is extant and has been published; see [10, 7:261; 16, 2:481–483] for references. I know of no other reference to al-Kāshi’s work Amthilaye a‘māl-e zīj; he may never have finished it.

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